

## ACTIVITY FIVE

### NEWTON'S SECOND LAW: CONSTANT MASS, CHANGING FORCE

#### PURPOSE

For this experiment, the Motion Visualizer (MV) is used to capture the motion of a cart moving along a flat, horizontal surface. The overall goal of this activity is for students to gain an understanding of the relationship between force, mass and acceleration. This will be accomplished by examining the change in acceleration that occurs when the applied force, exerted on an object, changes.

After this activity, students should be able to do the following:

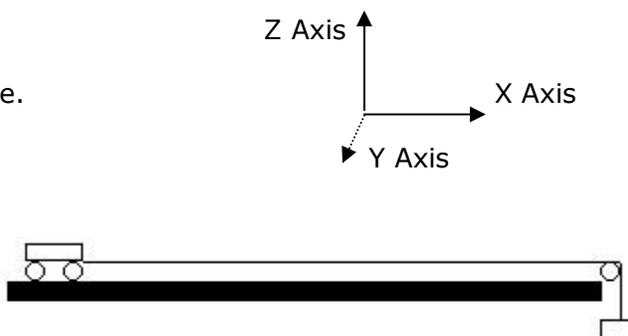
- ✓ Use words to describe Newton's Second Law of Motion.
- ✓ Explain how an applied force is related to an object's acceleration.
- ✓ Write out the equation for Newton's Second Law of Motion.
- ✓ Calculate the average acceleration of an object given distance and time data.
- ✓ Calculate the average acceleration of an object given force and mass data.
- ✓ Apply Newton's Second Law of Motion to various situations.

#### SOFTWARE SET-UP

This is a 2D, one-object experiment with horizontal motion. The distance from camera lens to plane of motion was set to 2.7 meters and the camera angle was set to  $-10^\circ$ . With this set-up, the software displays the horizontal motion on the X-axis and the vertical motion on the Z-axis.

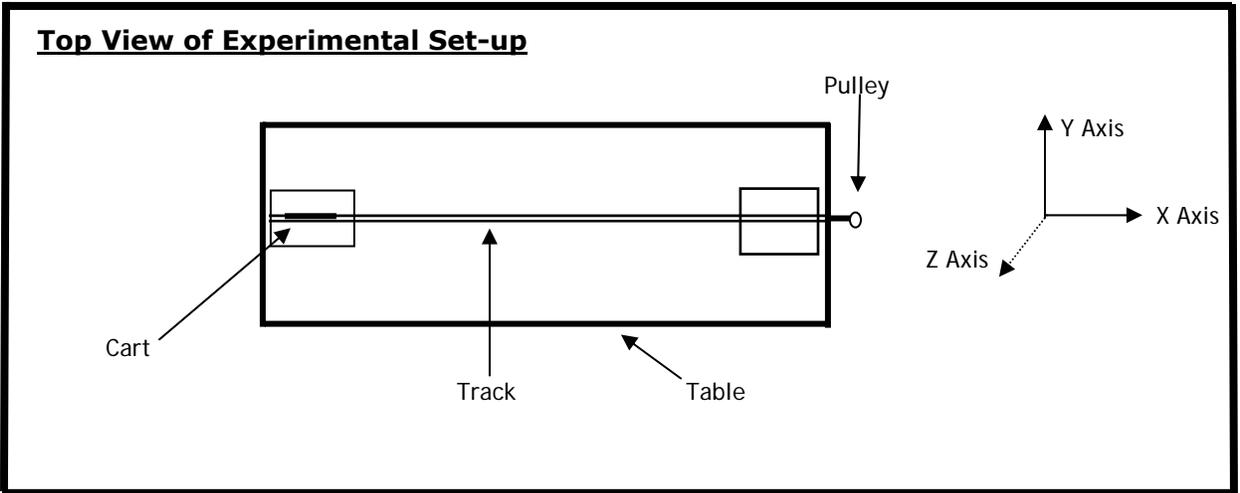
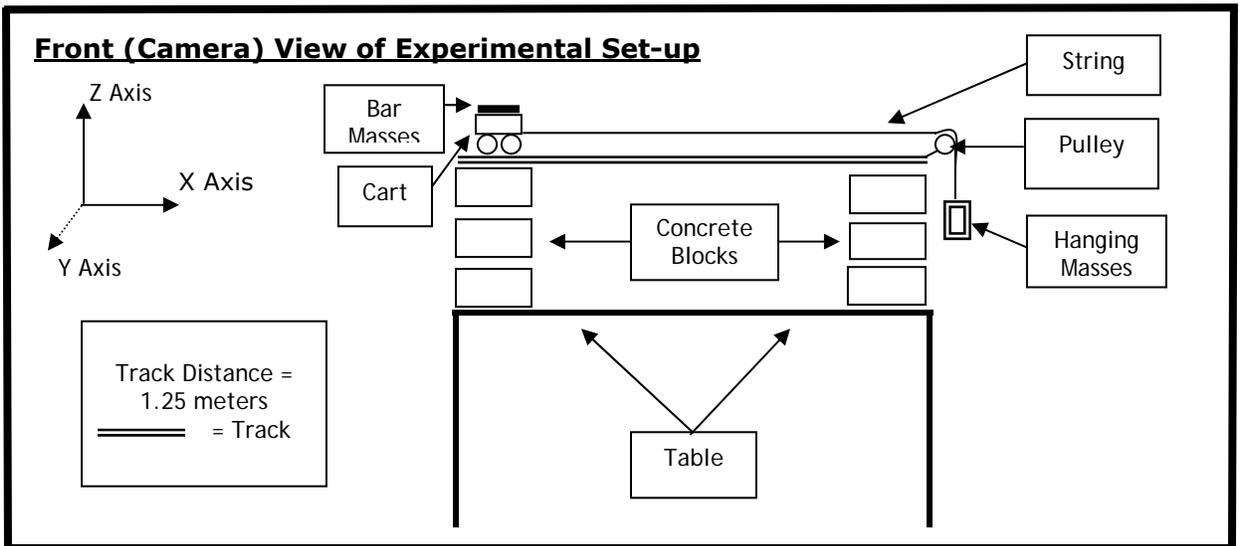
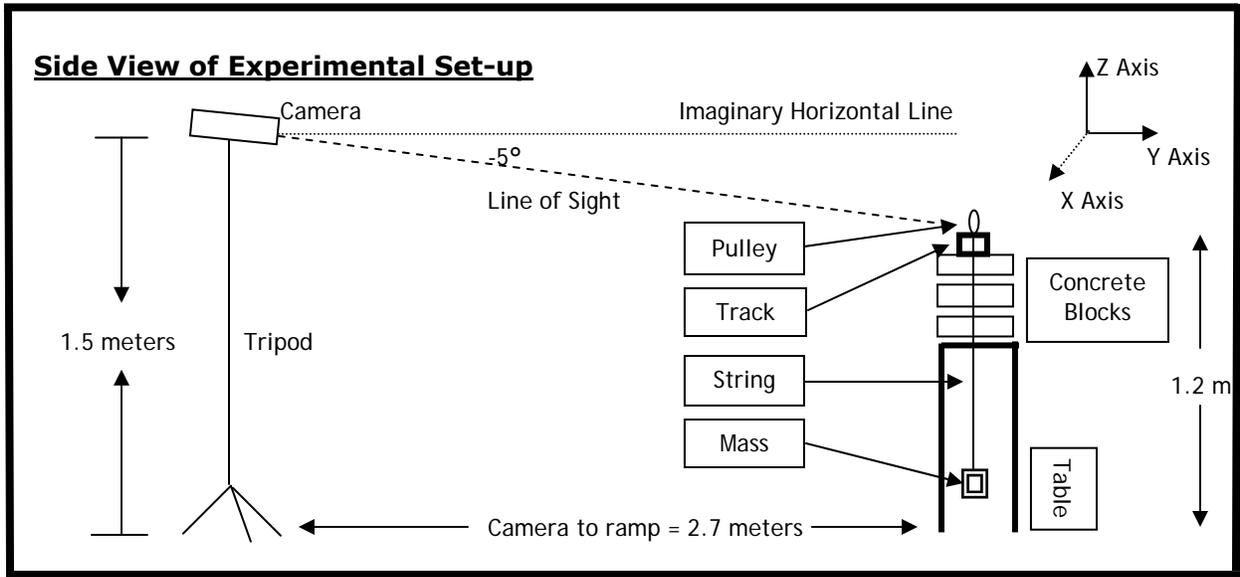
#### MATERIALS

- Low friction cart.
- 1 – 2 meter low friction track.
- Computer with MV software and hardware.
- Video camera with tripod.
- Angle measuring device.
- Tape measure or meter stick.
- String and paperclips.
- Sturdy object(s) to elevate track.
- Various hanging masses.
- Various bar masses.
- Pulley and clamp.



#### PROCEDURE

1. Use sturdy objects to elevate the ramp on both sides above the table.
2. Attach 1 meter string to cart.
3. Attach a paperclip to free end of string.
4. Attach a pulley to one end of the track.
5. Place cart at far end of track and put the string over the pulley with paperclip hanging.
6. Adjust the view finder of video camera to capture entire range of motion.
7. Use angle finder to determine camera angle. Enter this value in computer.
8. Measure distance from camera lens to plane of motion. Enter this value in computer.
9. Place cart at start point. Attach a mass to the paperclip.
10. Run experiment.



## DATA COLLECTION, PRESENTATION AND ANALYSIS GUIDELINES

In this activity, a string connected a cart and a hanging mass together. The hanging mass pulled the cart along the track in the X-direction, causing it to accelerate. Since the cart and the hanging mass are connected by a string, they move as one unit. Therefore, the acceleration experienced by the hanging mass will be the same as the acceleration experienced by the cart. The following scenario was analyzed:

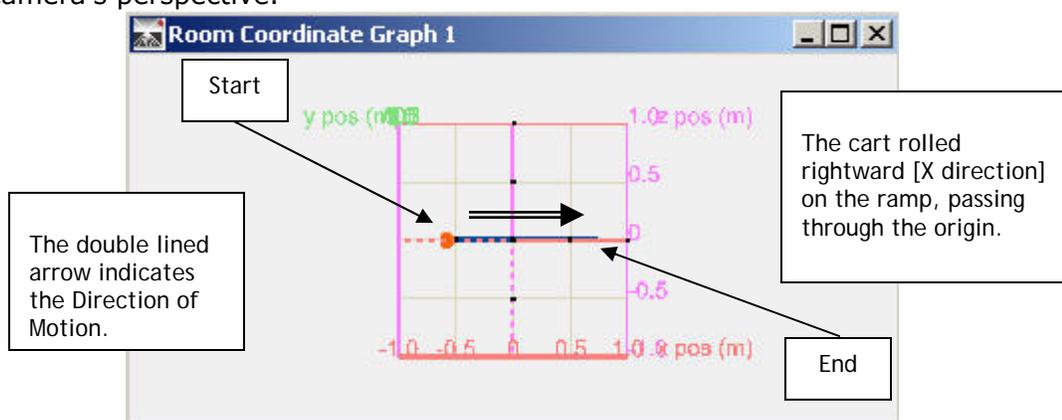
**The mass of the cart was constant while the hanging mass changed.**

- **Cart mass = 750g.**
- **Hanging masses = 10g, 20g, 50g and 100g.**

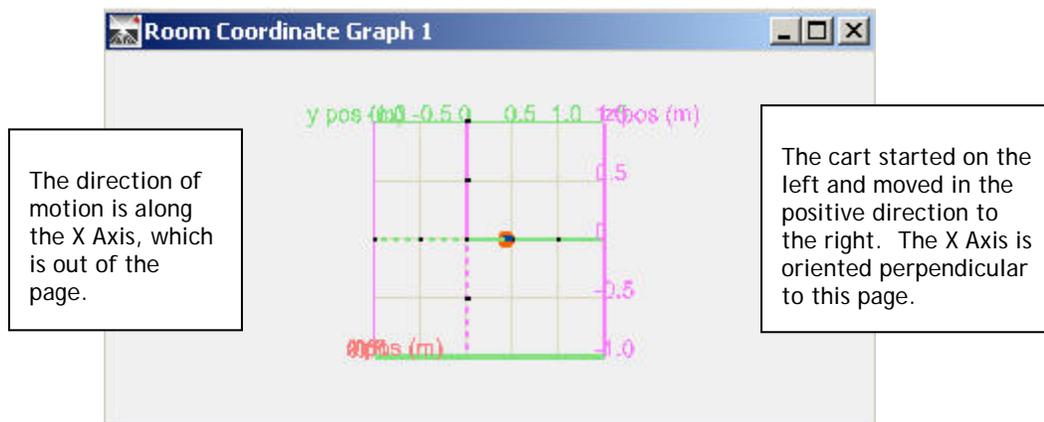
The following graphs and analyses are here to provide ideas about how the data can be used with students.

1. **Room Coordinate Graphs:** Images that show the actual path of the object in the X direction from various perspectives.

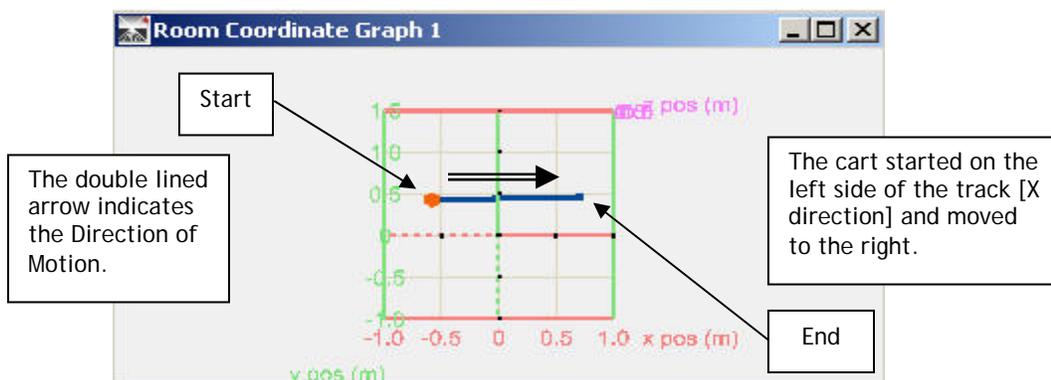
- **Front (Camera) view** – This view shows the motion of the cart from the camera's perspective.



- **Side view** – This view shows the motion of the cart looking from where the cart stopped moving to where it began moving.

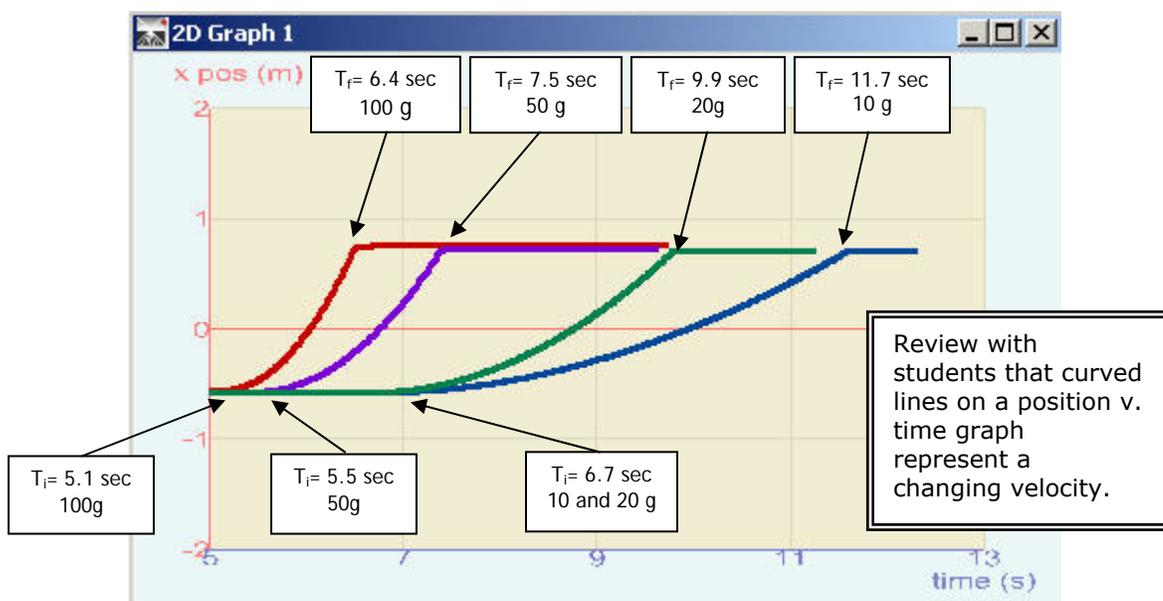


- **Top view (Bird's Eye) view** – This view shows the motion of the cart looking down on the table.



2. **X Position v. Time Graphs:** Graphical interpretations of the cart's displacement in the X-direction.

### X Position v. Time Graph: Constant Cart Mass (Cart Mass = 750g)



The cart covers the same distance (1.2 meters) in each trial. However, for heavier hanging masses, the time is shorter than for lighter masses.

### Cart Mass = 750 gram

Trial	Hanging Mass (grams)	Distance (meters)	Final Time (Sec)	Initial Time (sec)	Time = $T_f - T_i$ (sec)
1	100	1.2	6.5	5.1	1.4
2	50	1.2	7.5	5.5	2.0
3	20	1.2	9.9	6.7	3.2
4	10	1.2	11.7	6.7	5.0

From the above data, it is possible to calculate the acceleration that the cart experienced for each trial. This can be done by using the following equation:

$$\text{Distance} = \frac{1}{2} \text{Acceleration} \times \text{Time}^2$$

After rearranging, we have:

$$\text{Acceleration} = \frac{2 \text{ Distance}}{\text{Time}^2}$$

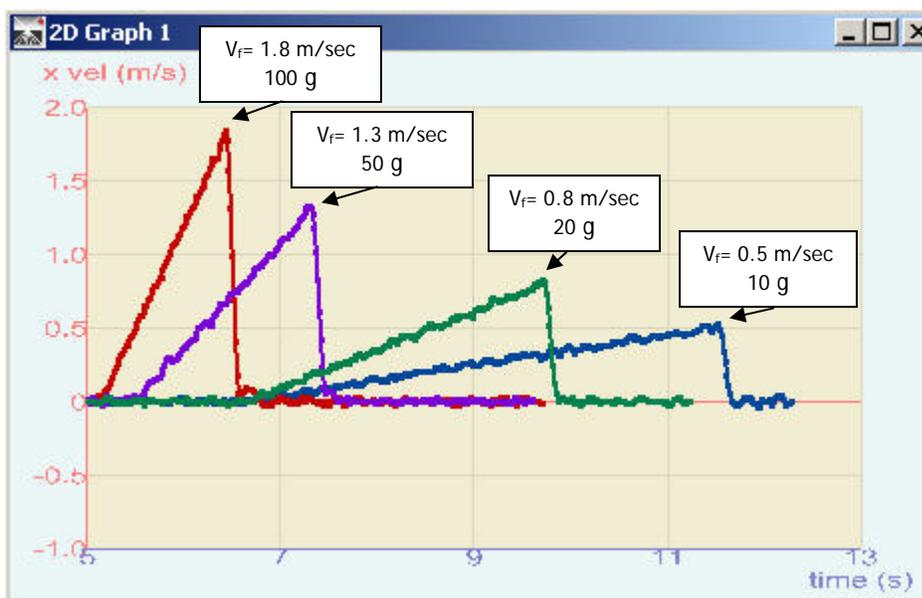
### Cart Mass = 750 grams

Trial	Hanging Mass (grams)	Distance (meters)	Time (sec)	Acceleration (m/sec <sup>2</sup> )
1	100	1.2	1.4	1.22
2	50	1.2	2.0	0.60
3	20	1.2	3.2	0.23
4	10	1.2	5.0	0.10

These acceleration values can be verified by calculating the slope on the Velocity v. Time graph of each trial or by analyzing the Acceleration v. Time graph of each trial.

3. **X Velocity v. Time Graphs:** Graphical interpretations of the cart's velocity in the X direction.

### X Velocity v. Time Graph: Constant Cart Mass (Cart Mass = 750g)



The time values for each trial above are the same as those taken from the X Position v. Time graph. The starting velocity for the cart was 0 m/sec for each trial. The acceleration for each trial can be determined by calculating the slope of each line.

$$\text{Slope} = \Delta Y / \Delta X$$

$$\Delta Y = \Delta \text{Velocity (m/sec)}$$

$$\Delta X = \Delta \text{Time (sec)}$$

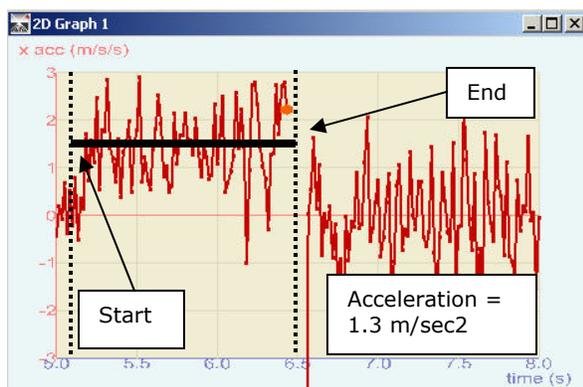
$$\Delta V / \Delta T = \text{Acceleration}$$

Trial	Mass (g)	Final Velocity (m/s)	Initial Velocity (m/s)	Time (sec)	Acceleration (m/sec <sup>2</sup> )
1	100	1.8	0	1.4	1.28
2	50	1.3	0	2.0	0.65
3	20	0.8	0	3.2	0.25
4	10	0.5	0	5.0	0.10

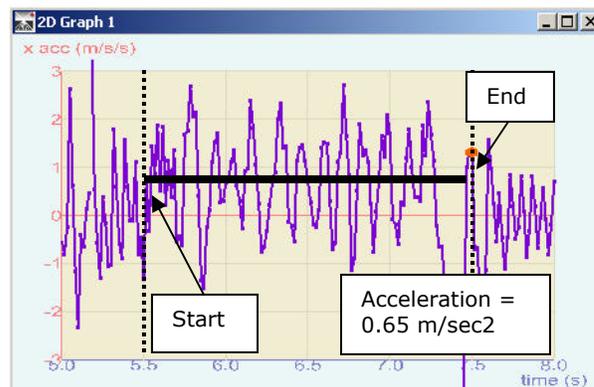
3. **X Acceleration v. Time Graphs:** Graphical interpretations of the cart's acceleration in the X direction.

### X Acceleration v. Time Graphs: Constant Cart Mass (Cart Mass = 750g)

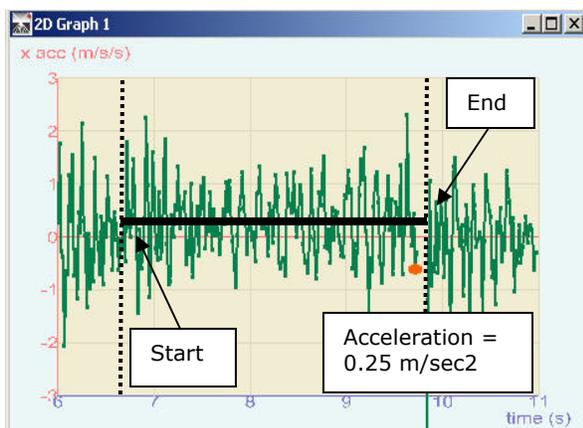
#### 100 gram hanging mass



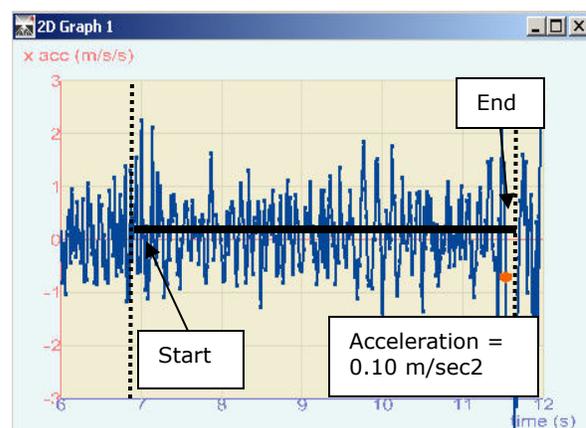
#### 50 gram hanging mass



#### 20 gram hanging mass



#### 10 gram hanging mass



Here is a comparison of the accelerations found by the three different methods:

Trial	Mass (grams)	$A = 2D/T^2$ (m/sec <sup>2</sup> )	$A = \Delta V / \Delta T$ (m/sec <sup>2</sup> )	$A_x$ v. T Graph (m/sec <sup>2</sup> )
1	100	1.22	1.28	1.3
2	50	0.60	0.65	0.65
3	20	0.23	0.25	0.25
4	10	0.10	0.10	0.10

The average accelerations, from the three methods above, are listed below:

Trial	Hanging Mass (grams)	Average Acceleration (m/sec <sup>2</sup> )
1	100	1.27
2	50	0.63
3	20	0.24
4	10	0.10

As the applied force decreases, the acceleration also decreases.

When the hanging mass is released, it supplies the force that accelerates the cart. If friction effects and the masses of the string and pulley are ignored, it is possible to estimate the mass that is being pulled [these values should **all** be approximately 750 grams.] This can be done by multiplying each hanging mass by 9.81 m/sec<sup>2</sup> to calculate its weight, the force exerted on the cart. Since the acceleration produced by each hanging mass is known, Newton's 2<sup>nd</sup> Law of Motion [ $F_{net} = ma$ ] can be applied to verify the cart's mass.

$$F_{net} = m_{cart}A$$

$$m_{cart} = F_{net} / A$$

Trial	Hanging Mass (grams)	Force Applied on Cart = Hanging Mass X "g" (N)	Average Acceleration (m/sec <sup>2</sup> )	Cart Mass (grams)
1	100	0.981	1.27	772.4
2	50	0.4905	0.63	778.6
3	20	0.1962	0.24	817.5
4	10	0.0981	0.10	981.0

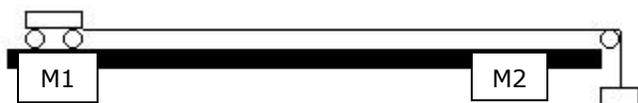
An error analysis can be completed to show how close the experimental values are to the theoretical values.

**Error Analysis:** % Error = (Experimental - Theoretical / Theoretical) \*100

Hanging Mass (grams)	Actual Cart Mass (grams)	Experimental Cart Mass (grams)	% Error
100	750	772.4	3.0
50	750	778.6	3.8
20	750	817.5	9.0
10	750	981.0	30.8

## NEWTON'S 2<sup>ND</sup> LAW APPLICATION

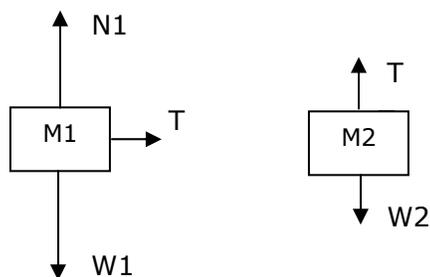
The scenario presented above is a classic Introductory Physics problem. Students are often asked to use Newton's 2<sup>nd</sup> Law of Motion to theoretically determine the acceleration of the system and/or the tension in the string. The data that is captured by the MV can be used to clearly show that the mathematical results found are, in fact, true. Below is an example of Newton's 2<sup>nd</sup> Law being applied to the above scenario to determine the acceleration of the system. Remember, since the cart and the hanging mass are connected by a string, they move as one unit. Therefore, the acceleration experienced by the hanging mass will be the same as the acceleration experienced by the cart. [Assumptions: no friction, the mass of the pulley and string are negligible.]



The role of the pulley is to simply change the direction of the string. It does not alter the tension in the string. The tension is caused by M2, which is hanging off the table.

Free-body diagrams:

$N_1$  = Support Force for Object 1  
 $T$  = Tension  
 $W_1$  = Weight of Object 1  
 $W_2$  = Weight of Object 2



We will apply Newton's 2<sup>nd</sup> Law to each object to solve for the acceleration in Trial 1.

**Trial 1: Cart Mass (M1) = 0.75 kg; Hanging Mass (M2) = 0.10 kg**

$$\Sigma F_x = T = M_1 A_{1x}$$

$$\Sigma F_y = N - W_1 = M_1 A_{1y}$$

$$A_{1y} = 0 \text{ m/sec}^2$$

$$A_{1x} =$$

$$T = M_1 A =$$

$$\Sigma F_x = 0 = M_2 A_{2x}$$

$$\Sigma F_y = T - W_2 = M_2 A_{2y}$$

$$A_{2x} = 0 \text{ m/sec}^2$$

$$A_{2y} =$$

$$T - W_2 = M_2 A$$

$$T = M_2 A + W_2$$

$$W_2 = M_2 "g"$$

$$W_2 = 100 \text{ g} \times 9.81 \text{ m/sec}^2$$

$$W_2 = 0.1 \text{ kg} \times 9.81 \text{ m/sec}^2$$

$$W_2 = 0.981 \text{ N}$$

$$M_1 A = M_2 A + 0.981 \text{ N}$$

Plug in known values and solve for A:

$$(0.75 \text{ kg}) A = (0.10 \text{ kg}) a + 0.981 \text{ N}$$

$$(0.65 \text{ kg}) A = 0.981 \text{ N}$$

<b>A Theoretical (Trial 1) = 1.51 m/sec<sup>2</sup></b>
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$$\% \text{ Error (Trial 1)} = (\text{Experimental} - \text{Theoretical} / \text{Theoretical}) * 100$$

$$\% \text{ Error (Trial 1)} = (1.27 - 1.51) / 1.51 * 100 = 16\%$$

<b>% Error (Trial 1) = 19%</b>
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This same analysis can be completed for each of the other trials. When this is done, the following values are found:

**Trial 2: Cart Mass (M1) = 0.75 kg; Hanging Mass (M2) = 0.05 kg**

$$A \text{ (Theoretical: Trial 2)} = 0.70 \text{ m/sec}^2$$

$$\% \text{ Error (Trial 2)} = 10\%$$

**Trial 3: Cart Mass (M1) = 0.75 kg; Hanging Mass (M2) = 0.02 kg**

$$A \text{ (Theoretical: Trial 2)} = 0.27 \text{ m/sec}^2$$

$$\% \text{ Error (Trial 3)} = 11\%$$

**Trial 4: Cart Mass (M1) = 0.75 kg; Hanging Mass (M4) = 0.01 kg**

$$A \text{ (Theoretical: Trial 4)} = 0.13 \text{ m/sec}^2$$

$$\% \text{ Error (Trial 4)} = 23\%$$

<b>Trial</b>	<b>Mass (grams)</b>	<b>A Experimental (m/sec<sup>2</sup>)</b>	<b>A Theoretical (m/sec<sup>2</sup>)</b>	<b>% Error</b>
1	100	1.27	1.51	19
2	50	0.70	0.63	10
3	20	0.24	0.27	11
4	10	0.10	0.13	23

## EXTENSIONS

- Collect data for changing cart mass and constant hanging mass.
- Collect data for two masses, connected by a string, that are hanging off either side of a pulley.
- Collect data for two masses, connected by a string, with the track at an angle rather than horizontal

## QUESTIONS

1. Suppose you are pushing your car down the street and then you double the amount by which you push your car. By how much does the acceleration of the car change? Explain.
2. How much force does a 20,000 kg spacecraft need to accelerate at  $2 \text{ m/sec}^2$
3. A 20 gram mass on a horizontal friction-free air track is accelerated by a string attached to another 20 gram mass hanging vertically from a pulley [like in this activity.] What is the force due to gravity in newtons of the hanging mass? What is the acceleration of the system?
4. A 5 kg mass resting on a smooth horizontal table is connected to a cable that passes over a pulley and is fastened to a hanging 10 kg mass. Find the acceleration of the two objects and find the tension in the string.
5. A 5 kg bucket of water is raised from a well by a rope attached to the bucket. If the upward acceleration of the bucket is  $3 \text{ m/sec}^2$ , find the force exerted on the bucket by the rope.
6. The data presented in the Force v. Acceleration graph below was collected by a group of Introductory Physics students. Analyze the data and determine the type of relationship that exists between Force and Acceleration. What does the slope on this graph tell us? Interpolate the acceleration if a force of 0.75 N was exerted on the object. Extrapolate the acceleration if the object experiences a 3 N force.

